

INVERSE PROBLEMS IN SOFT TISSUE ELASTOGRAPHY USING BOUNDARY ELEMENT METHODS

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Abstract

Elastography is an emerging functional imaging technique of current clinical research interest due to a direct relation between mechanical material parameters, especially the tissue stiffness, and tissue pathologies such as cancer. Digital Image Elasto-Tomography (DIET) is a new method that aims to develop elastographic techniques and create a simplified, improved breast cancer screening process. The elastic material information of breast tissue is reconstructed in the DIET concept from mechanically excited steady-state harmonic motion observed on the surface of the breast. While this inversion process has been traditionally approached using finite element methods, this surface-orientated problem is naturally suited to the use of Boundary Element Methods (BEMs) requiring the discretization only on the surface of the domain and on the interface of a potential inclusion. As only approximate information is available about breast tissue material parameters, this thesis presents the development of BEM based inverse problem algorithms suitable for the reconstruction of all material parameters in a proportionally damped isotropic linear elastic solid, where only the material density is known. The highly nonlinear identification process of a potential inclusion is treated through the combination of a systematic Grid-Search with gradient descent techniques. This algorithm is extended to a three-step algorithm that performs a background material parameter estimation before the subsequent identification of an inclusion and thus provides a confident indication for the differentiation between cancerous and healthy breast tissue. The development of these algorithms is illustrated by several simulation studies highlighting important reconstruction behaviors relevant to the elastographic inverse problem. A first experimental test on a silicon based breast phantom is presented.

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